Lecture 3

Unregulated Population Growth

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Outline

1. R 101 continued
   - Scripts
   - Parameters
   - Final word
2. Examples of unregulated population growth
3. Geometric growth model
4. Exponential growth model

R - scripts

We do not want to start writing each model from scratch every week.
We would like to save what we have done so far, and build upon it next time.
R-scripts are simple text files that contain a list of R commands. Basically they are computer programs.

1. Open New Script under the File menu.
2. This would create a blank window.
   First, we need to name and save the script file.
3. Click on the blank script window.
4. Then choose Save as. . . from the File menu.
5. Go to the directory you wish to save the file in, or create a new directory for your R-scripts and then choose it.
6. Once inside the directory, name the file hello.r and click on the save button.

R - scripts

7. In the blank script window type print("Hello World!")
8. Now save the file using Save option form File menu or pressing `Ctrl` + S.
9. Go back to the console / command window.
10. Type `getwd()` `Enter`
    This gives you the current working directory of R.
    You need change the working directory to the directory which contains the hello.r file.
11. In the File menu choose Change dir.
12. Then change the directory to the one in which you saved the hello.r file.
13. Verify that the working directory has changed by using the `getwd()` command again.

R - scripts

14. To run the hello.r program type
    `source("hello.r")` Enter

15. Another option is to choose the script window again and choose Run all from the Edit menu.

16. A third option is to run only some selected text within the script file or run only the line in which the cursor is currently on.
    This is achieved by selecting some text with the mouse or moving the cursor to the line you want to run, and then choosing "Run line or selection" from the Edit menu, or just pressing Ctrl + R.

17. After you have run hello.r and observed the output in the command / console window, close the hello.r script window.

R - scripts

18. Create a new script and save it with the name FirstPopModel.r
19. Write the following command lines into the script file:
    1. `Ninitial <- 10`
    2. `lambda <- 2`
    3. `N <- numeric(10)`
    4. `Time <- 0:9`
    5. `N[1] <- Ninitial`
    6. `N[2] <- lambda * N[1]`
    7. `N[3] <- lambda * N[2]`
    8. `N[4] <- lambda * N[3]`
    9. . . . (i.e., repeat the previous style of commands, each time incrementing the indexes)
    10. `N[10] <- lambda * N[9]`
    11. `print(N)`
    12. `plot( Time, N )`
20. Save the new commands that you added to the script file by pressing Ctrl + S

R - scripts

21. Now run the script file FirstPopModel.r

    Make sure you are in the right directory by switching back to the console window and using the `getwd()` command and Change dir from File menu if needed.

22. Observe the output in the console window and the graphical output.

23. Congratulations! You have now written and run your first population growth model in R.

Parameters vs. Variables

The recursion relation for the discrete-time growth model we just wrote is

$$N_{t+1} = \lambda \ N_t$$

- Obviously, N is the **variable** in this model
  1. Its value changes over time.
  2. We can attach a time index $(1, 2, \ldots, t, t+1,$ etc.) to each value .
- The value of λ, however, is fixed – does not change over time.
- λ is a **parameter** in this model – it participates in determining the dynamics of N, but its own value does not change over time.

Parameters vs. Variables

$$N_{t+1} = \lambda \; N_t$$

- The value of λ may be different in different populations or species or in different circumstances (different climate, temperature, pH, nutrient level, etc.).
- But for a single occurrence of the phenomenon (population growth) it is fixed.
- We can have as many parameters or variables as needed.
- For example, in the R-script we just wrote a second parameter is `Ninitial`, the initial population size.
- We usually write the parameters at the beginning of the R-script file, so we can use them later and easily locate and change their values.
- Try to change the values of `lambda` and `Ninitial` and observe how the output of the R program changes.

Final word

A good programmer is a lazy programmer.

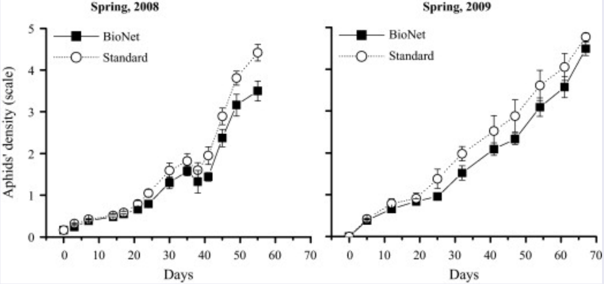
- We wrote an R-script, so not to write the entire model again and again each time we want to run it.
- We defined the parameter `lambda`, so we won't have to go over and change all the lines that calculate `N`, each time we want a different growth rate for the population.
- During the course, we will learn additional ways to make modeling in R easier, and the R-code more compact, so we can do a lot with just few lines of code.

Outline

The trouble with tribbles

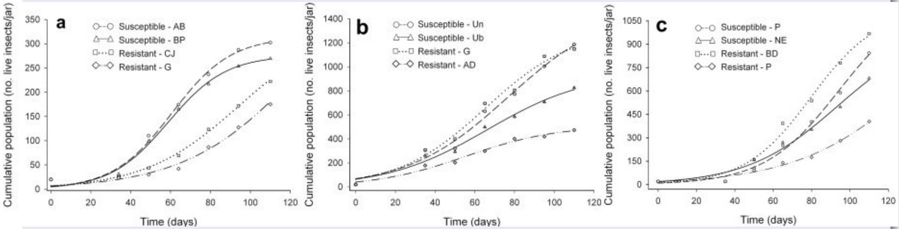
Pests

Agricultural pests; e.g., aphids



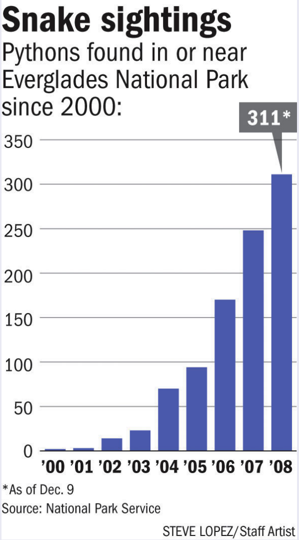
Pests

Stored-products pests



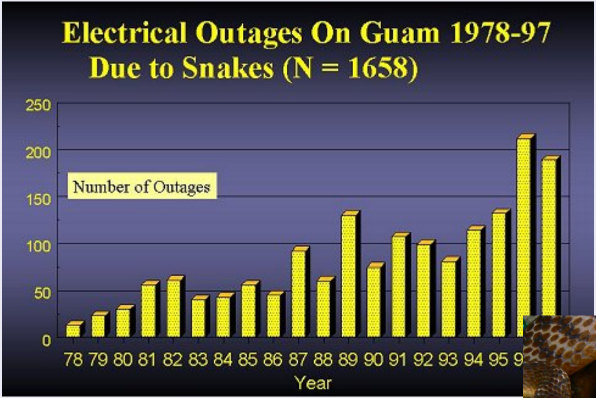
Invasive species

Burmese python in Florida



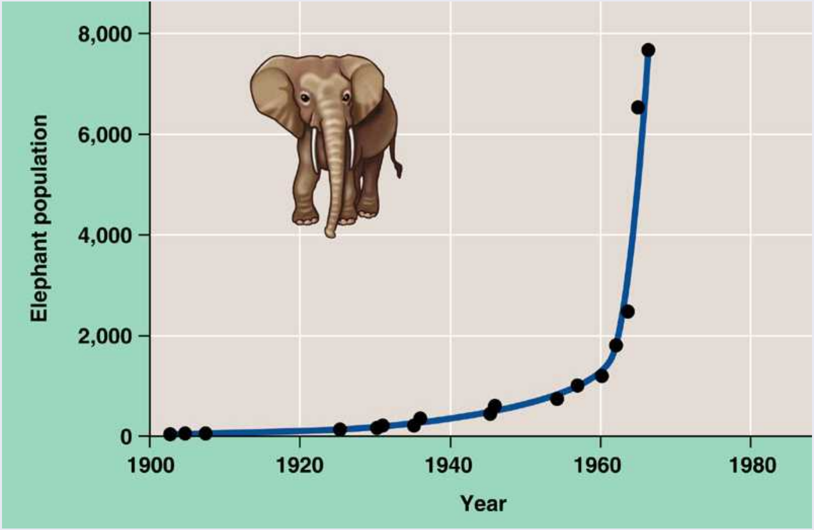
Invasive species

Brown tree snake in Guam



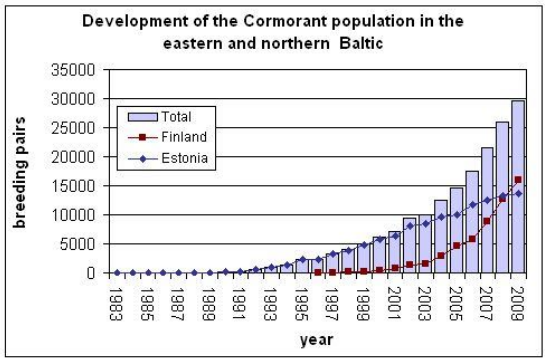
Recovering populations

Elephants in Kruger national park, South Africa



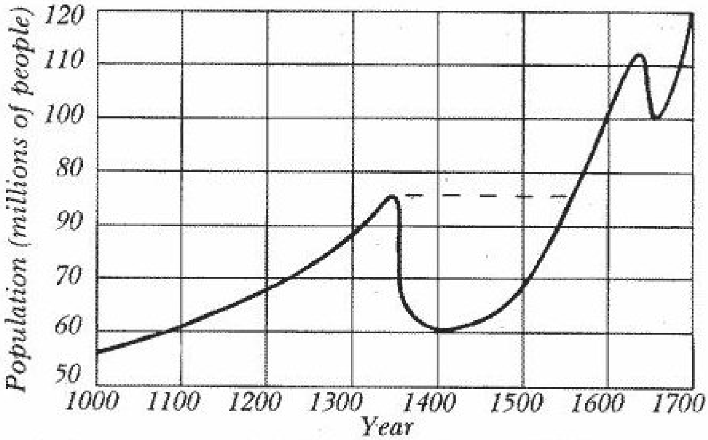
Recovering populations

Baltic sea cormorants



Recovering populations

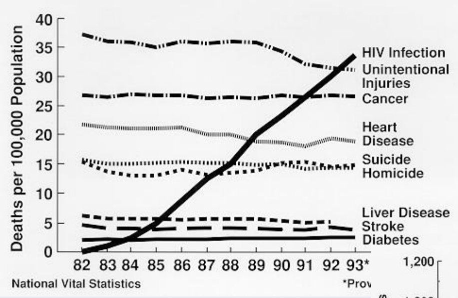
European population following the Black Death epidemic of 14th century

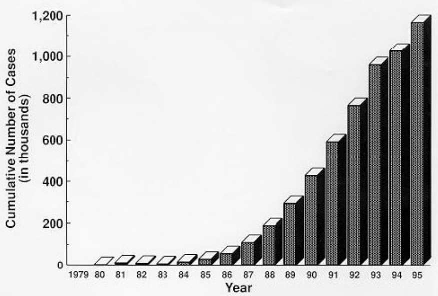


Recovery of European population following the plagues of 1347

Diseases and epidemics

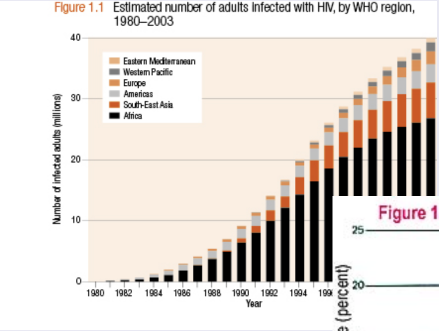
The AIDS epidemic

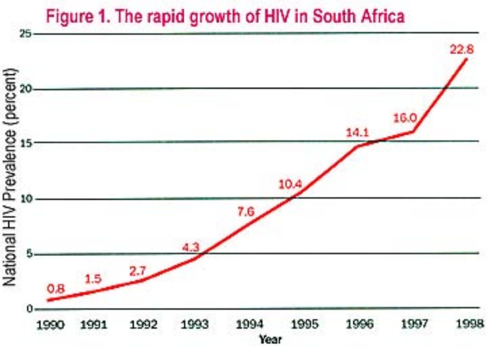




Diseases and epidemics

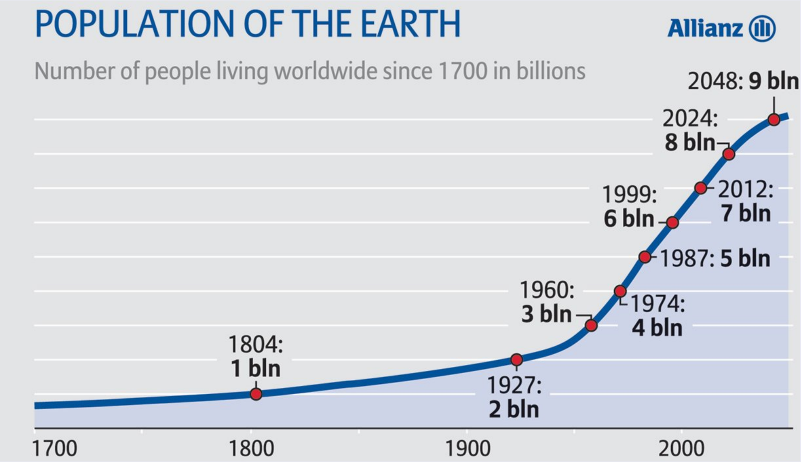
The AIDS epidemic in Africa





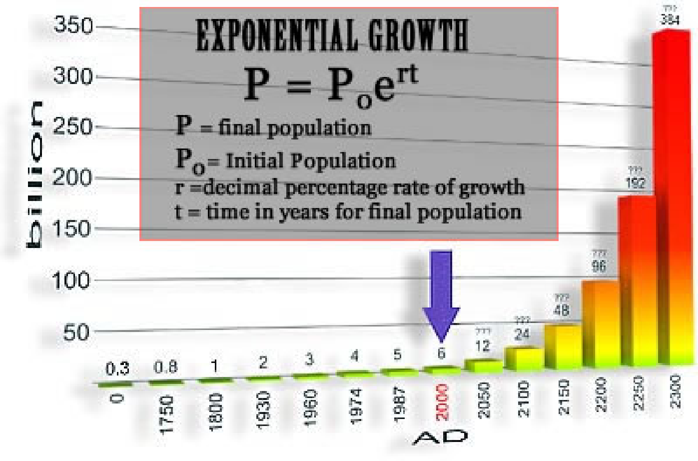
Human population growth

Global human population size



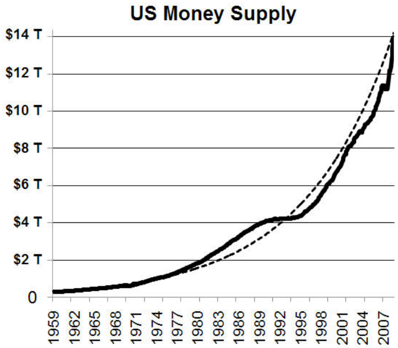
Human population growth

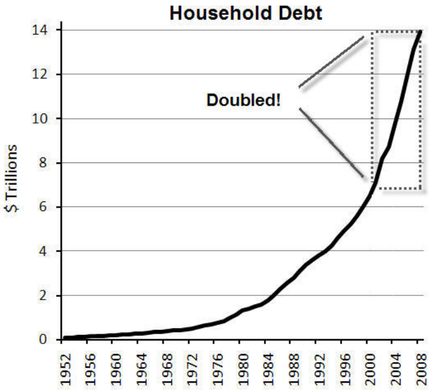
Global human population size: projected based on sustained unregulated growth



Human population growth

Exponential economic growth





Outline

Arithmetic model of population growth

Population increases by a fixed amount each time-step.

. . .

Arithmetic model of population growth

Why the arithmetic growth model is not a useful model?

1. Does not describe well the self-replicating nature of organisms.
2. Positive population growth, even if initial population size is zero – Spontaneous generation has been invalidated long ago.
3. Same growth rate for both small and large populations.
4. If negative, eventually negative population size is obtained – meaningless.

Arithmetic growth may apply when population grows mostly by means of immigration.

Geometric model of population growth

Population size is multiplied by a fixed factor time-step.

...

Geometric model of population growth

Recursion relation/Difference equation of geometric growth

$$N_{t+1} = \lambda \, N_t \;\Leftrightarrow\; \Delta N = (\lambda - 1) N_t$$

- λ is called **finite rate of increase**.
- It is the **average**/**mean** **per-capita** multiplication factor per one time-step.
- Average in the sense that some individuals contribute negative growth (die), some contribute positive growth (reproduce), some contribute zero growth (survive but do not reproduce). λ represents a weighted average of these different contributions.
- Classification of geometric growth:
  - $\lambda > 1 \Rightarrow \Delta N > 0$, population grows.
  - $\lambda = 1 \Rightarrow \Delta N = 0$, population size unchanged.
  - $0 \leqslant \lambda < 1 \Rightarrow \Delta N < 0$, population declines.

Geometric model of population growth

Why the geometric growth model a more useful model than arithmetic growth?

1. Directly related to the self-replicating nature of organisms.
2. $N_t = 0 \Rightarrow \Delta N = 0$.
3. Larger populations grow more (have higher ΔN; ΔN is proportional to N_t).
4. Even if growth is negative ($\Delta N < 0$; $0 < \lambda < 1$), negative values of population size are never obtained.

The net reproductive rate, R_0

- A second important parameter is the **net reproductive rate**, R_0, which is the **expected** lifetime reproductive output of a female.
- Example: for unicellulars, when time between divisions represents lifetime,
  - if there is no mortality, $R_0 = 2$.
  - if there is 25% mortality between divisions, $R_0 = 0.25 * 0 + 0.75 * 2 = 1.5$
  - if there is 50% mortality between divisions, $R_0 = 0.5 * 0 + 0.5 * 2 = 1$
  - etc.
- If we measure population growth in time-steps of 1 lifetime, $\lambda = R_0$.
- If in time-steps of 2 lifetimes, $\lambda = R_0^2$.
- If in time-steps of 3 lifetimes, $\lambda = R_0^3$.

The net reproductive rate, R_0

- Generalizing:

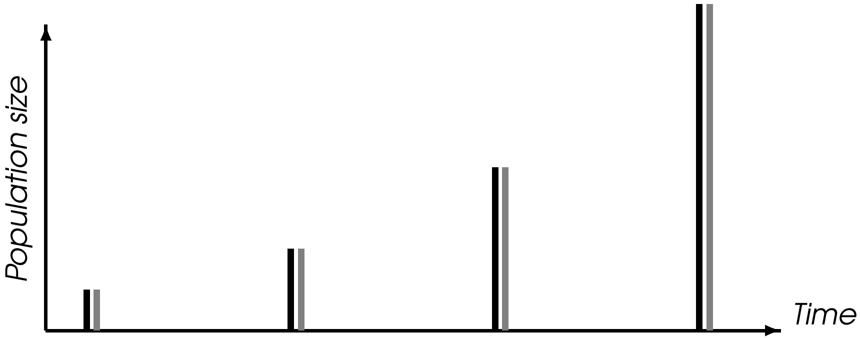
If T is lifetime / generation time,
and τ is the time-step of the model,
The relationship between λ and R_0 is

$$\boxed{\lambda = R_0^{(\tau/T)}}$$

- So, for example, if we measure population growth in time-steps of 10 generation times, $\tau = 10T$, then $\lambda = R_0^{10}$.
- But what about when $\tau < T$ we measure population growth in time-steps shorter than lifetime / generation time.
- Does this general relation still apply?

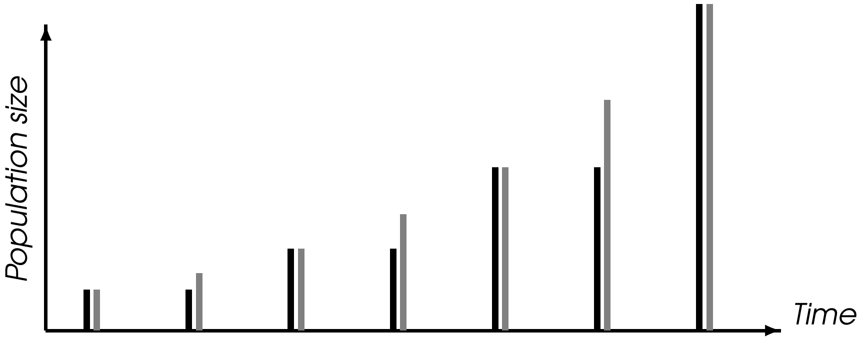
Synchronous and asynchronous reproduction

Synchronous reproduction: $\tau = T$.



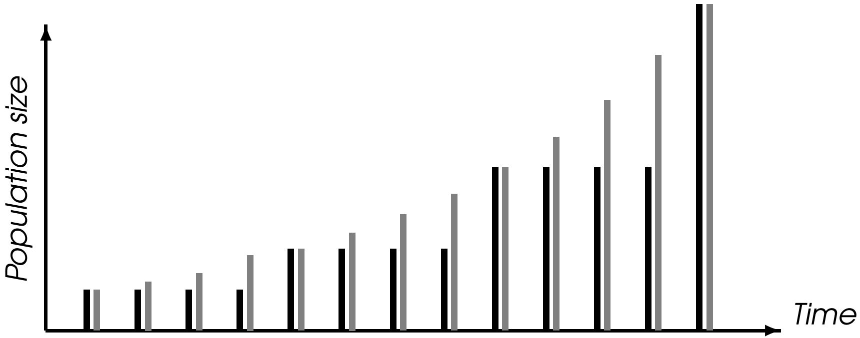
Synchronous and asynchronous reproduction

Synchronous reproduction: $\tau = T/2$.



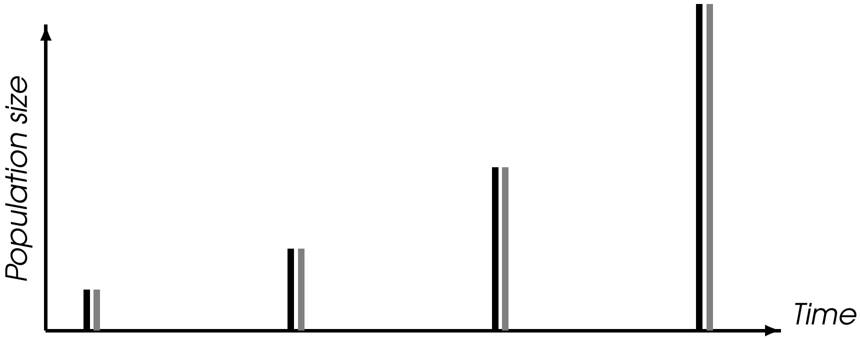
Synchronous and asynchronous reproduction

Synchronous reproduction: $\tau = T/4$.



Synchronous and asynchronous reproduction

But if we break the synchrony in reproduction.
For example, half the cells reproduce at noon and half at midnight.
Asynchronous reproduction: $\tau = T$.



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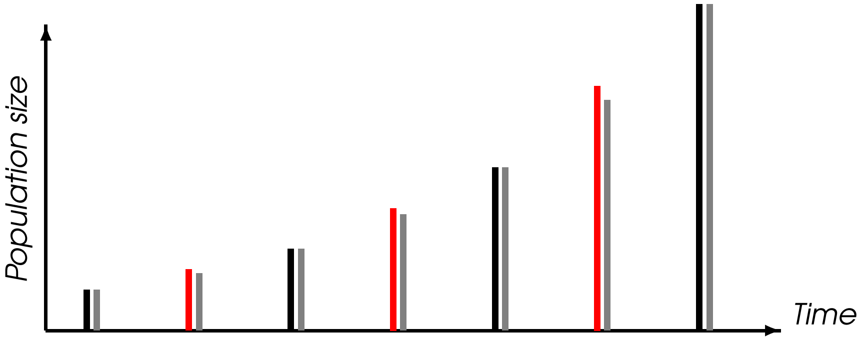

Synchronous and asynchronous reproduction

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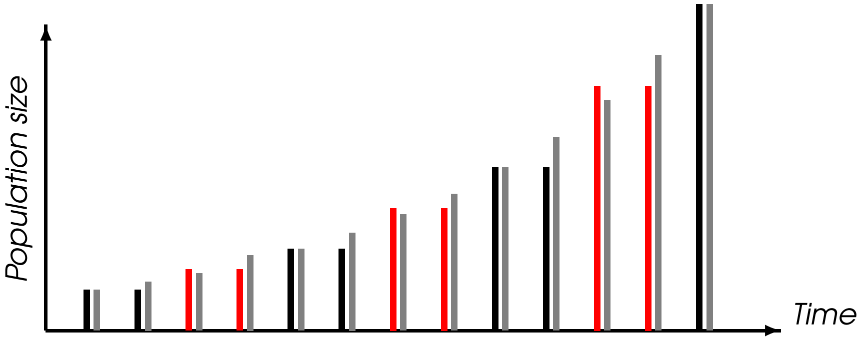
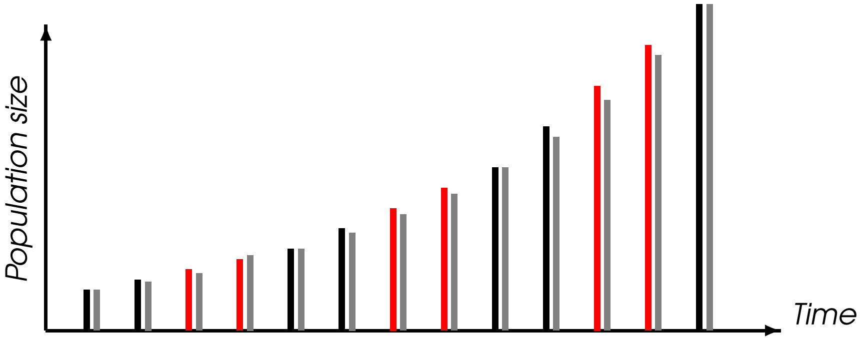


Synchronous and asynchronous reproduction

Even more asynchronous reproduction:
four "subpopulations", based on timing of division.



Outline

Exponential model of population growth

In the limit of completely asynchronous reproduction – reproduction events are continuously distributed over time – we obtain the **exponential growth** model.

- The population growth trajectory is given by
$$N(t) = N_0 e^{rt}$$
- The population growth rate is given by the differential equation
$$dN/dt = rN$$
- r is called **instantaneous rate of increase**.
- r has units of 1/time – unlike λ and R_0, which are pure numbers.

Exponential model of population growth

In the limit of completely asynchronous reproduction – reproduction events are continuously distributed over time – we obtain the **exponential growth** model.

- The three parameters are related according to

$$r = \frac{\ln \lambda}{\tau} = \frac{\ln R_0}{T}$$

- Classification of geometric growth:
  - $r > 0 \Rightarrow dN/dt > 0$, population grows.
  - $r = 0 \Rightarrow dN/dt = 0$, population size unchanged.
  - $r < 0 \Rightarrow dN/dt < 0$, population declines.